

anti-phase to the dynamic wind forces driving the structure. Structure motion is thus greatly reduced with the wind forces primarily driving the TMD instead of the structure. The energy of this motion is dissipated by the internal damping mechanism associated with the TMD.

The structure itself is modeled as a single degree of freedom mass-spring-damper system. The mass of this system is selected to give the same kinetic energy (at the TMD attachment point) as the structure in the vibration mode under consideration. This mass is known as the modal mass and, together with the calculated natural frequency of the vibration mode, defines the effective spring constant. The damping ratio for this system is either measured or known from prior experience with similar structures. It is typically in the range of 0.5% to 2.5% of critical damping.

The performance of the TMD can be expressed as a reduction in acceleration or dynamic deflection at some point on the structure or, equivalently, as an amount of damping added to the structure.

The optimum frequency and damping for the TMD are defined from the theory of systems for two degrees of freedom (Den Hartog, 1956; Crandall and Mark, 1963). These properties depend on the ratio of TMD to structure modal mass. The TMD mass is selected on the basis of maximum allowable TMD motion, minimum added damping required and off-tuned performance.

#### TMD Conceptual Design

Many different physical systems could be used to achieve the mass, natural frequency and damping properties required for the TMD. A number of design factors help to define the conceptual basis for the TMD design:

1. TMD natural frequency and damping should be tuneable on site to allow for variations in properties of the structure.
2. The TMD should work in all directions.
3. The TMD should require no external power for its operation.
4. The TMD should be able to operate at ambient outdoor temperatures.
5. The TMD should be operational at all times.
6. The design should incorporate means for braking and locking out the TMD.
7. The design should be simple, require a minimum number of components and be inexpensive to manufacture and maintain.

We have found that fully passive, pendulum-based designs with hydraulic damping are able to meet all of these design requirements. For towers, with relatively light weight and high frequencies, we have used both suspended and inverted (mass above pivot) spring-assisted pendulum designs. For high-rise

#### Tuned Mass Dampers for Towers and Buildings

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#### Abstract

Tuned mass dampers (TMD'S) are used to reduce the response of tall slender structures to resonant wind-induced oscillations. This paper reviews the authors' experience with the design and implementation of passive TMD's for a number of structures including:

1. The CN Tower in Toronto, Ontario: a free standing pre-stressed concrete communications tower.
2. Elevator towers in La Prade, Quebec: free standing steel lattice structures.

The paper describes the evolution of pendulum-based TMD designs and the variations required to accommodate the different types of structures.

#### Introduction

A TMD is a relatively small mass-spring-damper system attached near the top of a structure in order to reduce the resonant response of the structure to dynamic wind forces. Its natural frequency is tuned near the natural frequency of the structure's vibration mode that is to be controlled. When the structure begins to oscillate it excites the TMD into motion. The TMD inertia forces produced by this motion are approximately

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buildings, with much greater weight and lower frequencies, we have designed a cable supported pendulum system. For these designs, the natural frequency is tuneable by changing the pendulum length or the location of the springs relative to the pivot.

In all cases damping is provided by hydraulic cylinders attached at one end to the TMD mass and at the other end to the structure. As the mass moves relative to the structure, the hydraulic cylinder pumps oil around a closed loop hydraulic circuit that connects the two ends of the cylinder. Flow control valves in this circuit can be easily adjusted to change the resistance to motion and hence the damping in the TMD design. Separate shut down and isolation valves in the circuit perform the braking and locking functions.

#### TMD Design for the CN Tower

The CN Tower is a 460 metre pre-stressed concrete structure fitted with a 96 metre steel core antenna. Dynamic analysis and wind tunnel testing established the need for additional damping for tower modes involving primarily the antenna (CN Tower reports, 1974 & 1975); i.e. modes 2, 4 and 5 with corresponding frequencies of approximately 0.26, 0.79 and 1.1 Hz.

In order to effect the range of frequencies involved, it was decided to supply two TMD's located at approximately mid-span of the antenna and 15 metres apart. Space constraints on the design could reasonably be called severe.

The original specification of a 2700 kg mass with a maximum amplitude of 910 mm could not be met by any practical design. When the structural engineer (Nicolet & Associates, Montreal, Quebec) accepted a design that was three times heavier, this permitted a reduction in maximum amplitude also by a factor of three. This allowed us to develop practical pendulum based designs that would meet all design constraints.

The TMD's are comprised of a 9100 kg ring mass supported by three 1200 mm steel columns attached to the tower and ring by universal joints. Each support column is fitted with four mutually perpendicular horizontal springs and hydraulic cylinders. This arrangement provides for spring and damping forces with good radial symmetry. A bumper ring limits the maximum possible displacement of the mass to 375 mm.

For the lower TMD, the mass is suspended below the tower pivot and is tuned near the fourth mode with a natural frequency of 0.81 Hz. For the upper TMD, the mass is supported above the tower pivot and is tuned near the second mode with a natural frequency of 0.28 Hz. In both cases tuning is achieved by adjusting the location of the springs, nominally 300 mm from the tower pivot, relative to the pivot. In order to achieve a wide-band effect for

the various modes involved, the damping ratio is set relatively high at approximately 0.25 for both TMD's.

#### TMD Design for the La Prade Elevator Towers

The two identical elevator towers at the La Prade heavy water plant are free standing lattice structures with an inferior rectangular elevator shaft (Sacks, Lake and Cooper, 1979). Wind tunnel measurements indicated that vortex shedding from the rectangular shaft induced peak base bending moments that could exceed the design limit by a factor of three. A 1% mass ratio TMD was able to reduce these moments to less than one-half the design limit providing an effective cure for the problem.

We decided to develop a simplified generic TMD design based on the inverted pendulum since this design is more sensitive to spring location and inherently capable of lower frequencies without requiring very long pendulum lengths. The design would also be modular: the pendulum, springs and hydraulics would be contained in a steel frame so that the TMD could be transported and bolted in place as a complete unit. Thus space requirements and on-site assembly are minimized.

In order to provide redundant protection each elevator tower contains two of these modular TMD units in the machinery room atop the tower. Each pendulum mass is 2100 kg and is formed by stacking steel plates at the top of the support column. A plain spherical bearing forms the pivot point. Four mutually perpendicular horizontal springs are attached to the pendulum support column and to the frame. Two hydraulic cylinders are placed below two of the springs at right angles to provide the damping. A urethane bumper ring limits the maximum possible displacement of the mass. The complete unit is approximately 2.1 metres square by 2.0 metres high and contains a total mass of approximately 3450 kg.

At the design frequency of 0.73 Hz, the nominal position of the spring centreline is 460 mm above the pivot. The required tuning range of +/- 30% is obtained by changing the spring height above the pivot by +/- 130 mm.

Design specifications are as follows:

#### Elevator Tower and TMD Design Parameters

Total mass of elevator tower	280,000 kg
Vibration mode to be controlled	first bending
Natural frequency of vibration mode	0.775 Hz
Modal mass of tower at top	64,400 kg

buildings, with much greater weight and lower frequencies, we have designed a cable supported pendulum system. For these designs, the natural frequency is tuneable by changing the pendulum length or the location of the springs relative to the pivot.

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A typical design for a mass ratio of 0.02 could have a 400,000 kg steel mass block with dimensions 4m x 4m x 3m high and requiring a floor area of 9m x 12m. The design would use two 70 mm diameter cables per corner. A total of eight hydraulic cylinders (two per side) with eight separate hydraulic circuits would be used to provide the energy dissipation, braking and locking functions. The concept of the hydraulic circuit would be the same as for the tower TMD's.

#### Appendix I. - References

1. "CN Tower - Preliminary Antenna Performance Data for the Design of an Auxiliary Mass Damper", Boundary Layer Wind Tunnel Laboratory, University of Western Ontario, London, Ontario, March, 1974
2. "CN Tower - Final Evaluation of Tuned Mass Damper Performance", Boundary Layer Wind Tunnel Laboratory, University of Western Ontario, London, Ontario, February, 1975
3. Crandall, S.H. and Mark, W.D., Random Vibrations in Mechanical Systems, Academic Press, New York, N.Y., 1963.
4. Den Hartog, J.P., Mechanical Vibrations, Fourth Edition, McGraw-Hill, New York, N.Y., 1956.
5. Sacks, M.P., Lake, R.T. and Cooper, K.R., "Design of Tuned Mass Dampers for the La Prade Heavy Water Plant", Canadian Society for Civil Engineering, Structural Engineering Conference, Montreal, Quebec, June 7 & 8, 1979.